AMS 261: Probability Theory
Winter 2016

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Lectures: Tuesday, Thursday 4-5:45pm (J Baskin Engineering 169)
Office hour: Wednesday 2-3pm (or by appointment)

Course description and background: This graduate course offers an introduction to measure theoretic probability. Emphasis is placed on development of a background useful for stochastic modeling and methodological research in statistics. (Hence there is less emphasis on technical details from measure theory.)
Important math background includes basic concepts, methods and theory from real analysis, in particular, on sequences (and series) of real numbers, limits and continuity of functions, and sequences of functions (at the level of, say, chapters 3, 4, 5, 6 and 9 from Krantz, S.G., 1991, "Real Analysis and Foundations", CRC Press). Useful statistical background includes introductory probability theory and standard distribution theory (at the level of AMS 205B).

Grading: The course grade will be based on homework assignments, and (possibly) a take-home exam.

Course syllabus:
• **Probability spaces / Random variables**
Basic concepts and definitions for probability spaces. Sigma-fields, Borel sigma-fields and probability measures. Calculating probabilities: continuity of probability measure and Borel-Cantelli lemmas. Random variables. Induced probability measures. Distribution functions. Definition of the uniform distribution (Lebesgue measure) on the unit interval. Types of distribution functions.
• **Expectations**
Constructive definition of expectation as Lebesgue integral. Riemann-Stieltjes integral and expectations. Convergence theorems for expectations: Fatou lemma, monotone convergence and dominated convergence theorems. Uniform integrability. Inequalities for expectations.
• **Independence / Types of convergence**
Absolute continuity and the Radon-Nikodym theorem. Product probability spaces. Fubini theorem. Definition and properties of independence for random variables. Convergence of sequences of random variables. Modes of convergence and relations. Weak and strong laws of large numbers.

Reading/References: There is no required textbook. The lectures will be based on material taken from several books. The majority of the material is based on chapters 1–6 from:

• Fristedt, B., and Gray, L. (1997). *A Modern Approach to Probability Theory.* Birkhauser.

A few other books used (to a smaller extent) for the course notes are:

• Billingsley, P. (1995). *Probability and Measure* (Third Edition). Wiley.
• Rosenthal, J.S. (2006). *A First Look at Rigorous Probability Theory* (Second Edition). World Scientific Publishing Company.
• Grimmett, G., and Stirzaker, D. (2001). *Probability and Random Processes* (Third Edition). Oxford University Press.

The literature includes a large collection of textbooks/reference books on measure theoretic probability, several of them also including material on stochastic processes. Below is just a sample:

• Ash, R.B., and Doleans-Dade, C.A. (2000). *Probability & Measure Theory* (Second Edition). Academic Press.
• Chow, Y.S., and Teicher, H. (2003). *Probability Theory: Independence, Interchangeability, Martingales* (Third Edition). Springer.
• Chung, K.L. (2001). *A Course in Probability Theory* (Third Edition). Academic Press.
• Dudley, R.M. (2002). *Real Analysis and Probability* (Second Edition). Cambridge University Press.
• Durrett, R. (2010). *Probability: Theory and Examples* (Fourth Edition). Cambridge University Press.
• Gut, A. (2005). *Probability: A Graduate Course.* Springer.
• Shiryaev, A.N. (1996). *Probability* (Second Edition). Springer.
• Williams, D. (1991). *Probability with Martingales.* Cambridge University Press.